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Single-Frequency Quaternion Wave Equation

Vadim Sovetov*

Doctor of Technical Since, Russia

***Corresponding author:** Vadim Sovetov, Doctor of Technical Since, Russia, E-mail: sovetovvm@mail.ru

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Abstract

Using Maxwell's equations for a single-frequency quaternion, wave equations for magnetic and electrical intensity are obtained. It is shown that waves propagate in space in the form of circulation of these intensities. Since Maxwell's equations for the quaternion contain scalar parts in the form of electric charges of electrons and their spins (rotors), then along with magnetic and electric waves there are also electric and magnetic fluxes in the form of charges and rotors. The values of charge and rotor densities are calculated from magnetic and electrical intensities by means of their scalar product with the Hamiltonian operator in matrix representation.

According to Faraday's law, it is known that the derivatives of magnetic intensity correspond to electrical intensity, therefore magnetic and electrical waves can be considered as electromagnetic.

Keywords: Maxwell's equations; Quaternion; Electromagnetic waves; Circulation; Rotor

Introduction

It is known that wave equations are obtained from Maxwell's equations by means of the circulation operation from both parts of the equations for electric and magnetic intensities [1,2]. As a result of these operations, three Laplace equations are obtained for electrical intensity and three for magnetic intensity. For analytical, in particular, harmonic functions, we obtain equality to zero of all equations. Therefore, the equality of the Laplace operator to zero shows us that the wave functions must be analytic and harmonic.

At the same time, Maxwell's equations, based on the experiments of Faraday and Ampere, clearly show that the waves formed by the corresponding antenna currents must propagate in the form of circulations of magnetic intensity and electrical intensity [1,2]. In this case, the squares of the derivatives in the Laplace equation hide the phase relationships of the oscillations, so the magnetic and electric components of the waves in space are usually drawn in phase. But the circulation of tensions is formed by conjugate oscillations that differ from each other in phases.

Moreover, it was not clear from Maxwell's equations in 3D space why they did not contain particles, such as electrons, which create wave intensities. Therefore, there were assumptions that electromagnetic waves propagate in the ether. When deriving the wave equation, the assertion was used that the charge density and current density are equal to zero in a vacuum.

Maxwell's equations in 4D, obtained mathematically for the quaternion, showed that particles are formed from waves and are located on the 4th, scalar coordinate axis, which is not visible in 3D [3,4]. In this case, in a wave they are calculated as the scalar product of wave intensities with the Hamiltonian operator. The obtained equations satisfy the Cauchy-Riemann conditions taking into account the phase relationships of the waves [5]. Since functions based on hypercomplex numbers consist of conjugate functions, the Laplace operator is obtained by multiplying the conjugate Hamiltonian operators. Therefore, quaternion waves also satisfy the Laplace equation.

The purpose of this article is to derive wave equations based on the mathematically obtained Maxwell equations for a single-frequency quaternion.

Materials and Methods

Using the representation of the product of quaternions as a derivative of the conjugate quaternion with respect to time in the matrix representation **Q** by the quaternion function vector p, the equation is obtained [4]:

$$\mathbf{Q}^{\mathrm{T}}\mathbf{P} = T\begin{bmatrix}\partial_{s,t}p - \partial_{x,t}u - \partial_{y,t}v - \partial_{z,t}w \\ \partial_{x,t}p + \partial_{s,t}u + \partial_{z,t}v - \partial_{y,t}w \\ \partial_{y,t}p - \partial_{z,t}u + \partial_{s,t}v + \partial_{x,t}w \\ \partial_{z,t}p + \partial_{y,t}u - \partial_{x,t}v + \partial_{s,t}w\end{bmatrix} \Leftrightarrow \begin{bmatrix}\overbrace{\partial_{s,t}p - \nabla.\mathrm{f}(q)}^{\text{scalar}} \\ \partial_{x,t}p + \partial_{s,t}u - \overbrace{(\partial_{y,t}w - \partial_{z,t}v)}^{\text{circulation}} \\ \partial_{y,t}p + \partial_{s,t}v - (\partial_{z,t}u - \partial_{x,t}w) \\ \underbrace{\partial_{s,t}p}_{\nabla p} + \underbrace{\partial_{s,t}w}_{\partial_s f(q)} - \underbrace{(\partial_{x,t}v - \partial_{y,t}u)}_{\nabla\times f(q)}\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \qquad (1)$$

where $\nabla = [\partial_{x,t} \quad \partial_{y,t} \quad \partial_{z,t}]^{\mathrm{T}}$ – Hamilton operator in vector representation, $\mathrm{f}(q) = [u \quad v \quad w]^{\mathrm{T}}$ – vector of pure quaternions of functions of quaternion q.

This equation represents the Cauchy-Riemann Conditions (CRC) for a quaternion $q(t) = s(t) + ix(t) + jy(t) + kz(t)$, which corresponds to the law of conservation of energy of the 4D quaternion vector under functional transformations:

$$f[q(t)] = p[q(t)] + iu[q(t)] + jv[q(t)] + kw[q(t)] \quad (2)$$

A single-frequency quaternion $q(t)$ with a unit modulus $|q| = 1$ and a common circular rotation frequency ω_c can be represented in a polar coordinate system as follows:

$$q(t) = e^{\hat{i}\omega_c t} = \cos\omega_c t + \hat{i}\sin\omega_c t \qquad (3)$$

where $\hat{i} = (i + j + k)/\sqrt{3}$ – imaginary unit of a single-frequency quaternion.

Since equation (1) is based on the operation of multiplying quaternions, the solution to this equation is a quaternion or the corresponding function of a quaternion (2). These functions were used to describe magnetic and electrical strengths. As a function of intensity (2) one can use the sum of quaternions to different degrees [5].

Using the matrix representation of the quaternion, we write the magnetic intensity **H** as the product of the initial state vector **h** and the fundamental matrix to the power n [5]:

$$\mathbf{H}(n,\omega_c,t,\mathbf{h}) = \mathbf{\Phi}(n,\omega_c,t)\mathbf{h} = \left[\mathbf{E}\cos(n\omega_c t) + \hat{\mathbf{I}}\sin(n\omega_c t)\right]\mathbf{h} \quad (4)$$

where $\mathbf{\Phi}(\omega_c, t)$ - fundamental matrix, **E** – the identity matrix corresponding to the scalar part of the quaternion (3).

The imaginary part $\hat{i}$ of a quaternion is written in matrix representation through basis matrices **I, J, K**, as $\hat{\mathbf{I}} = (\mathbf{I} + \mathbf{J} + \mathbf{K})/\sqrt{3}$. $\mathbf{h} = [h_s \quad h_x \quad h_y \quad h_z]^{\mathrm{T}}$ – initial state magnetic intensity vector, h_s scalar part, $\mathrm{h} = [h_x \quad h_y \quad h_z]^{\mathrm{T}}$ – imaginary part of the magnetic intensity vector in the form of a pure quaternion. A similar expression was used for electrical intensity [4, 5].

Since we are considering a single-frequency quaternion with the same frequencies on imaginary coordinates in 3D space, all elements of the initial state vector **h** must be equal in magnitude. When normalizing the initial state vector to 1, the elements **h** will be equal to h=1/2.

We simplify expression (4) and write it as a vector:

$$\mathbf{H}(n,\omega_c,t,h) = h\begin{bmatrix}\cos(n\omega_c t) + \sqrt{3}\sin(n\omega_c t) \\ \cos(n\omega_c t) - \sin(n\omega_c t)/\sqrt{3} \\ \cos(n\omega_c t) - \sin(n\omega_c t)/\sqrt{3} \\ \cos(n\omega_c t) - \sin(n\omega_c t)/\sqrt{3}\end{bmatrix} \qquad (5)$$

Based on (1) for the magnetic intensity vector **H** and, accordingly, the pure quaternion H, the following two equations of the CRC for magnetic intensity were obtained [4]:

$\nabla . H = \rho_m$ – scalar equation, (6)

$\nabla \times \mathrm{H} = \partial_{s,t}\mathrm{H} + \nabla p_{\mathbf{H}}$ – vector equation. (7)

The corresponding two equations of the CRC are also obtained for the electric field strength:

$\nabla . E = \rho_q$ – scalar equation, (8)

$\nabla \times \mathrm{E} = \partial_{s,t}\mathrm{E} + \nabla p_{\mathbf{E}}$. – vector equation. (9)

Based on the CRC (1), which sets the requirements of the law of conservation of energy, it is shown that the elements of vector (5) are equal to 0 for both magnetic and electrical intensities [5]. Note that in the absence of a scalar part, the CRCs are not performed.

Technique of obtaining wave equations of single frequency quaternion

To test the CRC, we used expression (1), which was obtained by taking the derivative of a single-frequency quaternion with respect to the conjugate quaternion. To obtain the wave equation, we take the derivative of a single-frequency quaternion with respect to a quaternion. As a result, we obtain the following expression:

$$\boldsymbol{Qp} = \begin{bmatrix}\partial_{s,t}p + \partial_{x,t}u + \partial_{y,t}v + \partial_{z,t}w \\ \partial_{x,t}p - \partial_{s,t}u + \partial_{z,t}v - \partial_{y,t}w \\ \partial_{y,t}p - \partial_{z,t}u - \partial_{s,t}v + \partial_{x,t}w \\ \partial_{z,t}p + \partial_{y,t}u - \partial_{x,t}v - \partial_{s,t}w\end{bmatrix} \Leftrightarrow \begin{bmatrix}\overbrace{\partial_{s,t}p + \nabla.\mathrm{f}(q)}^{\text{scalar}} \\ \partial_{x,t}p - \partial_{s,t}u + \overbrace{(\partial_{z,t}v - \partial_{y,t}w)}^{\text{circulation}} \\ \partial_{y,t}p - \partial_{s,t}v + (\partial_{x,t}w - \partial_{z,t}u) \\ \underbrace{\partial_{z,t}p}_{\nabla p} - \underbrace{\partial_{s,t}w}_{\partial_s \mathrm{f}(q)} + \underbrace{(\partial_{y,t}u - \partial_{x,t}v)}_{\nabla\times \mathrm{f}(q)}\end{bmatrix} \qquad (10)$$

Expression (10) does not represent the CRC, so the elements of the vector are not equal to 0. Vector (10) consists of elements that form and emit an electromagnetic wave and elements that represent the wave itself, propagating in space. Compared to (1), the scalar parts are not subtracted but added. In the vector parts, the sign in the circulations was preserved, but the sign in the first sum changed to negative. When representing the difference ∇p and $\partial_s \mathbf{f}(q)$ the left side of the vector part (10) in the polar coordinate system as a circulation, this means that the direction of the gradient has changed. Consequently, the direction of the circulation currents $\nabla \times \mathrm{f}(q)$ along the three coordinates x, y, z began to coincide with the currents formed by the members ∇p and $\partial_s f(q)$ [5].

Magnetic wave equation

Let us consider expression (10) for magnetic intensity **H**.

$$\mathbf{W} = \begin{bmatrix} \overbrace{\partial_{s,t} p_{\mathbf{H}} + \nabla \cdot \mathrm{H}}^{\text{Gauss's law for H}} \\ \overbrace{\partial_{x,t} p_{\mathrm{H}}}^{\text{scalar } \mathbf{H}} - \overbrace{\partial_{s,t} u_{\mathrm{H}}}^{\text{Lenz's rule}} + \overbrace{(\partial_{z,t} v_{\mathrm{H}} - \partial_{y,t} w_{\mathrm{H}})}^{\text{circulation vector H}} \\ \partial_{y,t} p_{\mathrm{H}} - \partial_{s,t} v_{\mathrm{H}} + (\partial_{x,t} w_{\mathrm{H}} - \partial_{z,t} u_{\mathrm{H}}) \\ \underbrace{\partial_{z,t} p_{\mathrm{H}}}_{\nabla p_{\mathrm{H}}} - \underbrace{\partial_{s,t} w_{\mathrm{H}}}_{\partial_s \mathrm{H}} + \underbrace{(\partial_{y,t} u_{\mathrm{H}} - \partial_{x,t} v_{\mathrm{H}})}_{\nabla \times \mathrm{H}} \end{bmatrix} \quad (11)$$

As can be seen from (11), the scalar $\nabla.\mathrm{H}$ and circulation $\nabla \times \mathbf{H}$ are calculated from the magnetic intensity vector, so we will call this wave *magnetic*. We divide the vector (11) for magnetic intensity into two parts vertically. On the left side we write down expressions that are related to the magnitude of current in conductors. On the right side we write down expressions that are related to the magnitude of magnetic intensity.

In the scalar part on the left we write $\partial_{s,t} p_{\mathbf{H}}$ and on the right – the scalar product $\nabla.\mathrm{H}$:

$\partial_{s,t} p_{\boldsymbol{H}} \Rightarrow \nabla.\mathrm{H}$ – scalar equation, (12)

The arrow in the equation shows that a current in a conductor creates a current in space.

In the vector part on the left we write the derivatives $\nabla p_{\mathbf{H}}$ of along the coordinate axes *x, y, z* and the derivative $\partial_{s,t}\mathbf{H}$ with a minus sign. On the right we write down the circulation of magnetic intensity, which forms wave (spatial) currents:

$(\nabla p_{\mathbf{H}} - \partial_{s,t}\mathbf{H}) \Rightarrow \nabla \times \mathbf{H}$ – vector equation. (13)

Expression (12) corresponds to Gauss's law for rotors (electron spins) [4]. This law shows that the density of electric rotors at a given point in space corresponds to the scalar product of magnetic intensity with the Hamiltonian operator. It is known that the Lorentz force acts on moving charges in a magnetic field, "twisting" them and thereby turning them into rotors that create magnetic tension. Therefore, we determine the density of rotors by magnetic intensity.

However, according to Ampere's law, when magnetic intensity circulates in space, a spatial current $\nabla \times \mathrm{H} = \mathrm{J}$ is formed, which is the movement of spatial charges. Consequently, the rotors (electron spins) are transformed into electron charges, *i.e.* the circulation of magnetic tension "untwisting" the rotors by means of induction current. Thus, the magnetic field has inertia similar to the inertia of a body. In an oscillatory circuit, this role is performed by an induction coil. When performing CRC, the rotor (spin) densities are equal to the electron charge densities [5].

Expression (13) corresponds to Maxwell's equation for the circulation of magnetic intensity. The arrow in the expression shows that the left terms form a wave that propagates in the form of circulation of magnetic intensity in space in the directions *x, y, z*. In expression (13) $\nabla p_{\mathbf{H}}$ corresponds to the electric current *I* in conductors in the directions *x, y, z*.

Faraday established that a voltage is induced in a conductor when the magnetic flux Φ passing through it changes. The magnitude of the voltage *U* (electromotive force) is proportional to the rate of change of this flux. Mathematically, Faraday's law is written as $U = -d\Phi/dt$. Therefore, in (13) the expression $\partial_{s,t}\mathrm{H}$ can be replaced by the voltage *U* in the directions *x, y, z*:

$$(I + U) \Rightarrow \nabla \times \mathbf{H} \quad (14)$$

Current and voltage will form a circulation as complex conjugate functions of a quaternion. The wave propagates in the form of circulation of the vector of pure quaternion of magnetic intensity **H** in 3D space.

Thus, an antenna made of conductors forms a circulation of magnetic intensity in 3D space through the interaction of complex-conjugate values of current and voltage $(I + U)$ of quaternion, which, in turn, forms a spatial current from spatial electrons obtained through the scalar product $\nabla.\mathrm{H}$. Further, this current generates a circulation of magnetic intensity $\nabla \times \mathrm{H}$ (14), which, in the process of propagation in the form of conjugate waves, generates a spatial current of electrons, *etc.*

The process of "twisting" electrons in a magnetic field and converting them into rotors and the process of "untwisting" them during the circulation of the magnetic field and the occurrence of induction current slows down the oscillatory process similar to the inertia of a body or an induction coil. By analogy with an oscillatory circuit, the spatial current corresponds to the current in the conductor created by the electromotive force of a charged capacitor. On the capacitor plates we have gradients of voltages and currents that are perpendicular to each other and form circulations of magnetic intensity that create displacement currents [6].

In a vacuum, a wave propagates without loss of energy, since it satisfies CRC (6-9). In a free medium, the wave attenuates depending on ε_0 and μ_0.

In the 4D space of a quaternion, when it is depicted in 3D space with three imaginary coordinate axes, the real axis remains invisible and the values of the scalars in the form of a point mass are usually located at the ends of the intensity vectors that form them [3].

The wave has a harmonic form and changes its phase $\phi(t)$ during the change in time, as $\phi(t) = n\omega_c t = 2\pi f_c nt$, where n is the degree of the quaternion, f_c is the frequency of the carrier oscillation, $\omega_c = 2\pi f_c$ is the angular frequency of the carrier oscillation.

Unlike an oscillatory circuit, a wave generated by an antenna propagates in physical space over a distance r and changes its phase $\phi(r)$ in the process of propagation along a straight line. Let the harmonic wave have a length of λ meters. The wavelength corresponds to a change in the phase of the oscillation by 2π radians. The ratio of 2π to the wavelength is called the *wave number* and is denoted as $k = 2\pi/\lambda$ rad/m. The wave speed v represents the distance traveled per unit time. If the distance traveled by the wave is equal to the wavelength λ and the time is equal to the period of the wave T, then the speed is $v = \lambda/T$, hence $\lambda = vT$. As the degree of the quaternion n increases, the wavelength decreases by n times, therefore $kn = 2\pi n/\lambda$.

The change in the phase of the wave at a distance r will depend on the ratio of the distance traveled to the wavelength of the carrier oscillation:

$$\phi(r) = 2\pi nr/\lambda_c = k_c nr \qquad (15)$$

where $k_c = 2\pi/\lambda_c$ is the wave number for the carrier frequency with wavelength λ_c.

As already mentioned, the wave is constructed on coordinate axes with the same frequencies and propagates in vector space in space with distance coordinate axes. Over time, the frequencies of quaternion space will remain constant. Consequently, in the same time, the waves will travel the same distance in any direction of the spatial coordinate axes.

Since the waves propagate in the form of circulation around the origin point where the antenna is located, the initial point of propagation will be the origin of the 3D space, at the location of the antenna. As can be seen from expressions (10,11), circulation in 3D space can be divided into circulations in 3 planes YZ in the direction of the x axis, XZ in the direction of the y axis, XY in the direction of the z axis.

Based on equation (1), the CRC is performed for each vector element, so the wave can be considered in any direction along each *x, y, z* coordinate axis for the corresponding YZ, XZ, XY planes. Let us consider the propagation of a wave in the direction of the *x*-axis when dividing the scalar part along the 3 axes by 3 in power or $\sqrt{3}$ by amplitude.

From equation (11) we obtain expressions for the wave in the direction of the x-axis depending on the distance:

$$\boldsymbol{W}_x = \begin{bmatrix} \frac{\nabla.\mathrm{H}}{\sqrt{3}} \\ (\partial_{z,r} v_{\mathrm{H}} - \partial_{y,r} w_{\mathrm{H}} \end{bmatrix} \qquad (16)$$

Note that (16) corresponds to the wave equation for a plane wave in the *x* direction [2].

In accordance with the first element of vector (16) and the magnetic intensity vector (5) when changing the distance, the scalar product in the YZ plane is represented in the coordinate system of wavelengths (15) at *h*=1/2, as:

$$\frac{\nabla.\mathrm{H}}{\sqrt{3}} = \frac{1}{\sqrt{3}} [\partial_{x,r} \quad \partial_{y,r} \quad \partial_{z,r}] \begin{bmatrix} 0 \\ cos(nk_c r) - \frac{1}{\sqrt{3}} sin(nk_c r) \\ cos(nk_c r) - \frac{1}{\sqrt{3}} sin(nk_c r) \end{bmatrix}$$

$$= \frac{\sqrt{2}}{2} \partial_r \left(\cos(nk_c r) - \frac{1}{\sqrt{3}} \sin(nk_c r) \right) = -\frac{nk_c}{\sqrt{2}} \left(\sin(nk_c r) + \frac{1}{\sqrt{3}} \cos(nk_c r) \right) \qquad (17)$$

For comparison, we obtain the derivative with respect to distance of the scalar part of vector (5):

$$\partial_r p_{\mathbf{H}} = -\frac{nk_c}{2} \left(\sin(nk_c r) - \sqrt{3} \cos(nk_c r) \right) \qquad (18)$$

As can be seen from equation (12), the scalar part can also be obtained by the scalar product of the Hamiltonian operator with the magnetic intensity vector **H**:

$$\nabla.\mathrm{H} = -\frac{nk_c}{2} \left(\sqrt{3}\, sin(nk_c r) + cos(nk_c r) \right) \qquad (19)$$

Figures 1 show graphs of the scalar value of the magnetic intensity vector (5) with a change in the wave phase from the distance (15), obtained using formula (18) (red solid line) and formula (19) (red dotted line). The blue line shows the scalar product (17) for the *x* direction. The initial state vector has power 1.

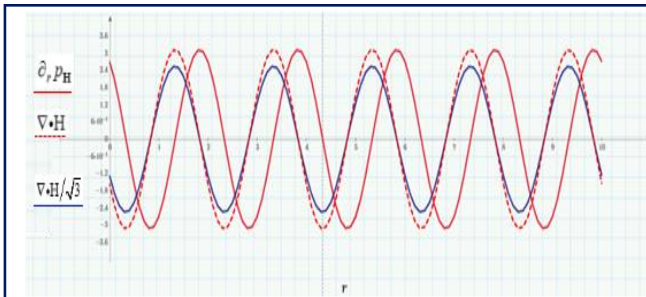

Figure 1: Graphs of scalars obtained from the scalar and vector parts of the magnetic intensity vector.

The powers of the scalars (red lines) $P_0 = M_0\pi^2/2$ are the same, but the phases differ by π/4. The power of the scalar part in the YZ plane in the x direction will be $P_x = M_x\pi^2/3$, *i.e.* the ratio of the powers of the scalars in 2D and 3D is 2/3. Constants M_0 and M_x are determined by parameters k_c and n. As the wavelength λ decreases and, consequently, the wave number k_c increases, the power increases. Also, the power increases with the increase of the quaternion degree n. At the same time, the ratio of powers in 2D and 3D remains the same. Since the wave scalars are calculated from the magnetic intensity of the wave by means of the scalar product with the Hamiltonian operator and their values are determined in 4D space on the scalar axis, they are "invisible" in 3D space. However, they exist and for a quaternion wave they can be calculated and, therefore, when they move in space in the form of a spatial current, a circulation of magnetic intensity is formed.

Figure 2 shows the density of the rotors of the magnetic wave intensity in the cross-section of the YZ plane with a change in the distance r from the antenna (green color). The circulation of magnetic intensity creates a flow of charges, *i.e.* an electric current, which is proportional to the magnitude of the circulation. In this regard, the graph in **Figure 2** shows the density of electric charges instead of the density of rotors. Red shows a positive charge and blue shows a negative charge. The charge density corresponds to the thickness of the shaded areas, *i.e.* the size of the point mass.

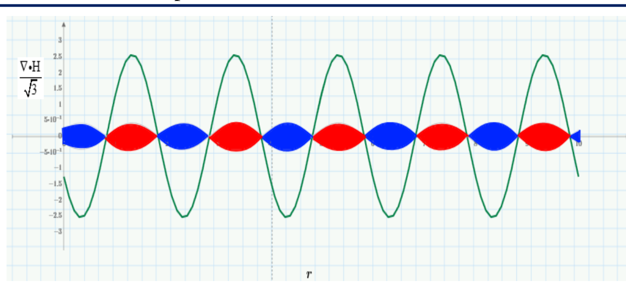

Figure 2: The magnitude of the rotor density of magnetic intensity or electric charges when changing the distance in the x direction.

Note that during the propagation of a wave no charge transfer occurs, the magnitude of the charge at a given point in space is simply related to the magnetic intensity and is determined using the scalar product of the intensity with the Hamiltonian operator.

Let us calculate the circulation of magnetic intensity on the YZ plane for the direction of wave propagation x using the formula $(\partial_{z,r}v_{\mathrm{H}} - \partial_{y,r}w_{\mathrm{H}})$ from (16). The expressions for the magnetic intensities along the y and z axes are presented as elements of the intensity vector in (5) when the distance r changes. Note that the axes of the spatial coordinate system y and z are orthogonal and therefore when calculating derivatives one of the axes should be rotated by π/2.

Using formula (16), we obtain the expressions:

$$\partial_{y,r}w_{\mathrm{H}} = \frac{nk_c}{2}\left(\frac{1}{\sqrt{3}}\sin(nk_c r) - \cos(nk_c r)\right) \quad (20)$$

$$\partial_{z,r}v_{\mathrm{H}} = \frac{nk_c}{2}\left(\sin(nk_c r) + \frac{1}{\sqrt{3}}\cos(nk_c r)\right) \quad (21)$$

Figure 3 shows the circulation of magnetic intensity in the form of a circle (green) and the circulation gradient in the form of arrows (brown) [20,21]. **Figure 4** shows the circulation of magnetic intensity (green) depending on the distance traveled by the wave and its projection onto the horizontal plane (red) and onto the vertical plane (blue). If we consider the YZ plane as complex, then the horizontal plane corresponds to the change in tension for the real axis and the vertical plane corresponds to the change in tension for the imaginary axis. According to (14), the circulation of a wave for magnetic intensities can be considered as an electromagnetic wave.

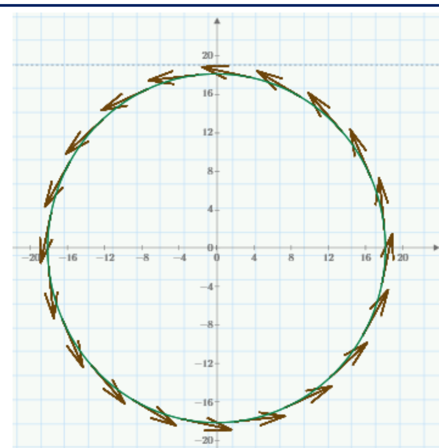

Figure 3: Circulation of magnetic intensity on the YZ plane and its gradient.

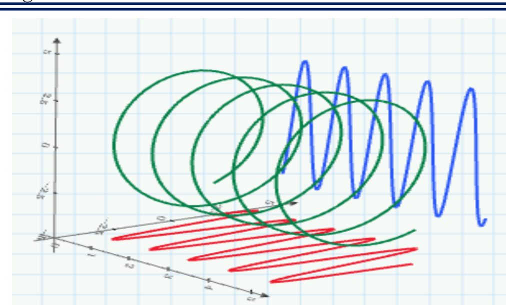

Figure 4: Circulation of magnetic intensity depending on the distance and projection of circulation on the horizontal and vertical planes.

The radius of the circumference of the described circle, shown in **Figures 3 and 4**, is calculated as $R = A_1 k_c hn$, where A_1 is the amplitude of the elements of the magnetic intensity vector, taking into account the division of the scalar part by power equally into three imaginary axes. The area of the described circle is calculated as $S = \pi R^2$.

Thus, it is shown that the magnetic wave propagates in space in the form of circulation of magnetic intensity in 3D vector space. The fourth scalar coordinate of the quaternion is not visible in 3D space, but the value of the rotor density, which corresponds to the density of electric charges, can be calculated from the projections of the intensity onto the orthogonal coordinate axes. The projections of the circulation of magnetic intensity onto the horizontal and vertical planes correspond to the magnetic intensity described by the vector of a single-frequency quaternion, taking into account the shift of π/2 of the coordinate axes. The change in magnetic intensity on the real axis, according to Faraday's law, can be interpreted as a change in electrical intensity and the presented wave can be considered as electromagnetic.

Electric wave equation

Let us consider expression (10) for electrical intensity **E**.

$$\mathbf{W} = \begin{bmatrix} \underbrace{\overbrace{\partial_{s,t} p_{\mathbf{E}} + \nabla \bullet \mathbf{E}}^{\text{Gauss's law for E}}}_{\rho_q} \\ \overbrace{\partial_{x,t} p_{\mathbf{E}}}^{\text{scalar E}} - \overbrace{\partial_{s,t} u_{\mathbf{E}}}^{\text{EMF of self-induction}} + \overbrace{(\partial_z v_{\mathbf{E}} - \partial_y w_{\mathbf{E}})}^{\text{circulation vector E}} \\ \partial_{y,t} p_{\mathbf{E}} - \partial_{s,t} v_{\mathbf{E}} + (\partial_x w_{\mathbf{E}} - \partial_z u_{\mathbf{E}}) \\ \underbrace{\partial_{z,t} p_{\mathbf{E}}}_{\nabla p_{\mathbf{E}}} - \underbrace{\partial_{s,t} w_{\mathbf{E}}}_{\partial_{s,t} \mathbf{E}} + \underbrace{(\partial_y u_{\mathbf{E}} - \partial_x v_{\mathbf{E}})}_{\nabla \times \mathbf{E}} \end{bmatrix}. \quad (22)$$

As can be seen from (22), the scalar $\nabla.\mathrm{E}$ and circulation $\nabla \times \mathbf{E}$ are calculated from the electric intensity vector, so we will call this wave *electric*.

The previous reasoning for a magnetic wave is applied to waves formed by an electric field strength E and we obtain the following equations:

$\partial_{s,t} p_{\boldsymbol{E}} \Rightarrow \nabla.\mathrm{E}$ – scalar equation, (23)

$(\nabla p_{\mathrm{E}} - \partial_{s,t} \mathrm{E}) \Rightarrow \nabla \times \mathrm{E}$ – vector equation. (24)

Expression (23) corresponds to Gauss's law for electrical intensity, which determines the charge density in space based on electrical intensity. From equation (24) it is evident that electric waves propagate in the form of circulation of electric intensity, *i.e.* in the form of spatial particles of rotors that form a magnetic flux.

Similarly, in an oscillatory circuit, the gradients of electrical intensity and currents change on the capacitor plates, resulting in the formation of an electric field circulation and the formation of a displacement current. That is, on one plate of the capacitor, electrons are "twisted" into rotors, transferred between the plates in the form of rotors (magnetic wave) and on the other plate with the opposite sign they are "untwisted" and transferred along the wire in the form of charges [6].

Regardless of whether we consider the circulation of electrical intensity or magnetic intensity, waves still appear when they interact. Therefore, Maxwell's equations for a quaternion have the same form. Therefore, the formulas and figures presented above will be the same when replacing magnetic intensity with electrical intensity and the wave will be electromagnetic.

If the dimensions of the wave source in 3D are small enough in relation to the distance and the energy from it spreads uniformly in all directions, then the source can be considered a point and the wave diverging from it will be spherical. In this case, the energy emitted by the source is evenly distributed over the entire surface of the wave sphere.

When the radius is doubled, the surface area increases fourfold. The energy transferred by a wave through a cross-section with an area of 1 m2 in a time of 1 s, *i.e.* the power transferred through a single cross-section, is called the *wave intensity*. Thus, the intensity of a spherical wave decreases inversely proportional to the square of the distance from the source.

Conclusion

The wave equations are obtained by taking the derivative of a single-frequency quaternion with respect to a quaternion in matrix representation. The wave equations are divided into two parts, the first part acts as an antenna and forms a wave, the second describes the wave in space. It is shown that, depending on the intensity under consideration, waves can be divided into magnetic and electrical. Waves propagate in space in the form of circulation of magnetic and electrical intensities. The scalar parts of the waves in the form of electron charges and their spins (rotors) are not visible in 3D space and are calculated by the scalar product of the corresponding intensities with the Hamiltonian operator. Since, according to Faraday's law, a change in magnetic intensity corresponds to a change in electrical intensity, these waves can be considered as electromagnetic.

The wavelength, like the processes in an oscillatory circuit, depends on the duration of the process of converting rotors into electron charges during the circulation of magnetic intensity and the process of converting electron charges into rotors during the circulation of electrical intensity.

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